A mixed-integer linear programming approach for energy-constrained mobile anchor path planning in wireless sensor networks localization

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Abstract In sensors localization, it is not economically argumentative to equip many static nodes with GPS. The most widespread solution is to utilize from mobile location-aware nodes called mobile anchors. A substantial amount of research to propose mobile anchor trajectory to improve localization accuracy, localization latency, network coverage and traversed path has been reported. However, none of the existing static mobile anchor path planning mechanisms has emphasized on increasing mobile anchor lifetime and reliability that is necessary due to limited energy. In this paper we propose a novel mobile anchor trajectory planning scheme called Optimal Priority based Trajectory with Energy Constraint (OPTEC) in order to address these issues. The proposed scheme utilizes the Mixed Integer Linear Programming optimization (MILP) approach for optimal route planning in the presence of location uncertainty for deployed sensors. Several important evaluation metrics including localization coverage, localization success, ineffective beacon points and energy-location uncertainty product are also defined for a comprehensive comparison of static mobile anchor trajectory plans. In this paper, static sensors utilize a range-free localization algorithm. Simulation results reveal that the proposed mobile anchor trajectory planning approach can surpass other existing trajectories in terms of localization error, mobile anchor energy consumption and even sensors lifetime.

Keywords—Energy efficiency, Localization, Mobile anchor, Range-Free localization, Static path planning, Wireless sensor networks

1. Introduction

Wireless sensor networks (WSNs) consist of a large number of small size, low-cost and low-power sensors. They are broadly deployed in numerous applications including military, health, environment monitoring, object tracking, security, surveillance and so on [1]. In many circumstances such as tracking or event-detection functions location awareness of sensor nodes is necessary. Moreover, the location of sensors is required for network operations such as energy saving routing mechanisms and topology control. Manually configuring location information into each node during deployment is not an option. Similarly, equipping every node with a Global Positioning System (GPS) receiver fails because of cost and deployment limitations [2]. Hence, localization, is one of the major issues in WSNs. The most promising solution to the localization problem is to equip a small number of sensor nodes with GPS or to install them at predetermined locations. These sensors that are called static anchors or reference nodes broadcast their location coordinates, called beacons. Location-unaware sensors, called unknown sensors, use the anchor’s coordinates to estimate their locations [3],[4]. They use beacons to extract information about their geometric or connectivity relationship with anchors depending on the localization approach being applied.

Location estimation of unknown sensors in WSNs is done in two different ways based on whether ranging information is used or not: range-based and range-free approaches [5]. In range-based approaches, unknown sensors use range information to estimate their positions by either “Triangulation” or “Trilateration/Multilateration” depending on the signal features used. However, range-free approaches use topological information instead of range information [2], [6]-[8]. In these approaches, it is assumed that a sensor is in reach of an anchor whenever it is located within a circular area of known radius and center determined by the anchor’s transmission power and it’s coordinates, respectively. Once an unknown node has received beacons from all of its neighbor anchors, it can determine that it is located in the intersection of the communication circles of these anchors. The intersection area is representative of localization accuracy both for the estimative region of the sensor and its estimated position. The challenge for range-based approaches arises when range measurements are not accurate. The intuitive solution to this problem is increasing the number of static anchors. In the range-free approach, more anchors allow a smaller intersection area and hence improve localization accuracy.

Therefore, in WSNs, accurate location estimation of sensors in

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both range-free and range-based approaches demands a large number of static anchors, which adds to localization cost [9]. A compendium of knowledge in the recently published papers in [9]-[16] state that utilizing a mobile anchor can solve the aforementioned problem. It moves across the region of interest where the unknown sensors are deployed and broadcasts beacons. It acts like virtual static anchors.

A literature survey on mobile anchor assisted localization indicates that two fundamental issues to investigate are efficient localization algorithms and optimum beacon trajectories. Nevertheless, considerable research attention has been attracted to mobile anchor path planning mechanisms in recent years. Most of them have considered location accuracy, localization latency, path length and full coverage as critical metrics to determine mobile anchor trajectory. Even though path length is representative of mobile anchor energy consumption, none of the existing mobile anchor static path planning mechanisms consider energy conservation as a primary concern. However it is considered as a main concern in this paper because of the following reasons:

- Mobile anchor energy is not infinite in disaster management applications such as localization of deployed sensors for fire detection in woods.
- In dynamic environments, it is probable that the location of the sensors change. Therefore, periodical localization, which makes energy conservation a main concern, is sometimes necessary.
- In practice, localization approaches do not remove location uncertainties instantly. They gradually reduce them each time the mobile anchor transmits a beacon and that is another reason for which we prefer to keep the anchor alive as long as possible.

A novel path planning mechanism is proposed in this paper for an energy-constraint mobile anchor named Optimal Priority based Trajectory with Energy Constraint (OPTEC). It considers mobile anchor energy consumption as the primary concern. The key motivation for presenting an energy-constrained mobile anchor path planning mechanism is to extend the lifetime of a mobile anchor.

It is assumed that the mobile anchor knows the estimative region of the deployed sensors with different location uncertainties. This paper aims at developing an efficient mobile anchor path planning mechanism, based on defined priorities for considered beacon locations. It utilizes the Mixed Integer Linear Programming (MILP) optimization approach to determine the optimal beacon points and construct the shortest path among them.

In summary, the contributions of this paper are as follows: First, we define energy constraint of the mobile anchor as a new important challenge in mobile anchor guiding approach. Second, we prioritize feasible beacon locations based on the number of sensors in their vicinity and their contribution to reduce the uncertainty of sensor location. Then, we introduce a priority based MILP optimization approach to find the optimal path in the presence of an energy constraint. In addition, we define new essential evaluation metrics that include localization coverage, localization success, ineffective beacon points rate and energy-location uncertainty product to provide a fair comparison of static path planning schemes in range-free mobile anchor-assisted localization task.

The remaining part of this paper is organized as follows. Section 2 describes existing mobile anchor trajectories. For each trajectory, description of its basics is given, followed by a brief qualitative discussion on their pros and cons. Section 3 details the proposed trajectory planning algorithm while Section 4 illustrates the simulation setup, evaluation metric and compares the performance of the proposed trajectory with existing ones via simulation results and then the conclusion is given in Section 5.

2. Related works

Localization with low cost and high accuracy is a fundamental issue in resource-constrained WSNs. The idea of mobile anchor assisted localization scheme presented by Sichitiu and Ramadurai [17] is to eliminate the requirement of deploying a large number of static anchors in resource-constrained WSNs. There has been a large body of research on mobile anchor assisted localization algorithms both in static and dynamic sensor networks over the last decade. WSNs with static sensors are explored in this paper because this group of sensor networks promises a wide spectrum of application scenarios including monitoring tasks and military applications. Utilizing a cost-effective localization algorithm, and finding an efficient mobile anchor trajectory are the two main tasks in mobile anchor-assisted localization of WSNs. In this section, we review some related works.

2.1 Mobile anchor localization

Many approaches for obtaining per-node location knowledge have been explored in mobile anchor localization algorithms. Just like other localization algorithms, these approaches can be categorized into range-free and range-based methods based on the type of knowledge utilized for location determination. This paper surveys approaches from both of these two categories. Range-free algorithms estimate sensor node’s coordinates or its estimative region using connectivity information between sensor nodes and anchors without ranging (i.e., distance or angle) information. Localization based on Sequential Monte Carlo (SMC) method [18], localization based on convex method [19], localization based on geometric constraints [20] and localization based on a perpendicular bisector of a chord [21],[22] are examples of range-free methods that have been presented. Range-based techniques consist of two steps: (i) range measurement based on Time of Arrival (ToA) [23]-[25], Time Difference of Arrival (TDoA) [26],[27], Angle of Arrival...
Mobile anchor path planning

Mobile anchor guiding mechanisms proposed in recent research papers are classified into two groups: static and dynamic trajectory planning. In the static category like the ones proposed in [29]-[34], the mobile anchor path is determined beforehand. In other words, the mobile anchor follows a predefined and deterministic path. However, in dynamic trajectory planning [35],[36], the mobile anchor determines the trajectory on the fly based on the real distribution of the sensor nodes. Due to the dynamic behavior of WSNs (e.g., random sensor node dropping and their movement by wind or animals), dynamic path planning provides better localization accuracy, but at the expense of increased localization latency. This paper focuses on static path planning schemes for delay sensitive applications.

The RWP mobility model presented in [37] is a random planning approach that became common because of its simplicity. However, it leads to non-uniform coverage of the network field. Koutsonikolas et al. [32] proposed three static trajectories for the mobile anchor node, called SCAN, DOUBLE SCAN and HILBERT. In SCAN, the mobile anchor moves along one dimension (either along the y-axis or x-axis) and the distance between two parallel and adjacent segments of the trajectory defines the resolution. SCAN is effective for uniform distributed sensors and guarantees that all the sensors are able to receive beacons if the correct resolution is used. However, this simple and easily implemented trajectory suffers from collinearity of beacon points which degrades the localization accuracy. DOUBLE SCAN is proposed to obviate this issue by guiding the mobile anchor along both x and y dimensions. This advantage does not come for free, but in exchange for the doubled trajectory length compared to the simple SCAN, for the same resolution. HILBERT is another static trajectory that guides the mobile anchor along HILBERT space-filling curve. This mobile anchor trajectory provides more non-collinear beacon points by making more turns in mobile anchor trajectory without significantly increasing the path length. The main drawback of this curve based trajectory is the poor localization of the sensors located near the border, since nodes at the borders receive fewer beacons, only from one direction, rather than the central nodes.

Circular based trajectory called CIRCLES that is proposed in [33] defines an alternative to mitigate the collinearity that appears in straight line based trajectories.

In the case of square-like deployment areas, CIRCLES experiences lack of coverage at the four corners of the sensing field. However, coverage imperfection can be smoothed by expanding the diameter of the concentric circles. Naturally, this results in path length extension and consequently increases the energy consumption of the mobile anchor.

One of the beneficial static trajectories is LMAT. The LMAT algorithm is proposed in [34] by Han et al based on the idea of equilateral triangle configuration which was initially proposed in [29]. The mobile anchor moves along an equilateral triangle trajectory and transmits the beacons at triangle vertices. LMAT proposes an optimal trajectory of a mobile anchor based on an equilateral triangle to achieve higher localization accuracy and full coverage while decreasing the number of beacon points and non-collinear beacon points. This path shape suggestion is due to the fact that the location uncertainty of unknown nodes is the least when the trajectory of mobile anchor node forms an equilateral triangle, as demonstrated in [29].

Another static trajectory that has been recently proposed by Rezazadeh et al. in [30] is called Z-curve. The mobile anchor trajectory is built based on Z-shaped curves. The authors have indicated two reasons for considering Z-shaped curves. The first one is that such a trajectory has short jumps to overcome the collinear beacons problem and the second one is that it creates a superior path for transmitting three consecutive non-collinear beacons in order to reduce the localization latency. They have also presented an obstacle-handling trajectory to tolerate and manage the presence of obstacles in the field.

To the best of our knowledge, MILP optimization approach hasn’t been used for mobile anchor path planning problem before. So, we decided to model the mobile anchor trajectory planning as MILP problem in this paper. Optimal beacon points selection similarities with optimal sensors deployment in WSNs motivated us to investigate and review some sensors deployment solutions have been proposed in [38]-[41] which have utilized MILP approaches. Joint energy efficient routing and node placement optimization algorithm, JR-SPEM, which is solved using mixed integer programming modeling introduced in [38], reduces energy consumption in structural...
health monitoring WSN to prolong the network lifetime. Directional sensors placement has been modeled by MILP in [39] whose goal is to minimize the number of directional sensors that need to be deployed to monitor a set of discrete targets in a sensor field. Guo et al. in [40] also study the linear sensor placement problem in monitoring oil pipelines with the goal of maximizing the network lifetime. To achieve this goal the authors have investigated equal-power placement schemes which have been formulated by MILP approach as part of their work. Sensor placement solution which has been proposed by Berry et al. in [41] has used MIP to detect maliciously-injected contaminants while minimizing the expected fraction of population at risk for an attack. The MILP modeling approach has been used in our work have commonalities with MIP approach in [41]. The first one is graph representation which has been used to model the respective problem in both papers. Furthermore, binary decision variable which is introduced as the final solution, pre-calculated parameters, priority of beacon points in our work and probability of attack in [41], which are involved in objective function definitions and considering limited amount of energy and limited number of sensors as main constrains in our work and [41], respectively, are other similarities.

3. The proposed mobile anchor path planning scheme

3.1 Network environment and assumptions
In this paper, a large number of stationary sensor nodes that communicate with each other wirelessly (WSN) are randomly deployed in an $L \times L$ outdoor area. Moreover, one GPS-enabled mobile anchor can be utilized in the localization task. Assume that the mobile anchor knows a priori the information of initial estimated regions of these sensors. The area size of the estimated region of each sensor is defined as the sensor location uncertainty. The deployed sensors might have different location uncertainties. For simplicity, a circular region with a known center and radius or the overlap region of two circles with given centers and radius indicate the sensors’ initial estimative regions (IER) as explained in Figure 1(a) and (b).

respectively. It worth mentioning that, if IERs of sensors were not known, a pre-localization step would be added and hereinafter the mobile anchor would be aware of the IERs. Each of basic movement trajectories such as RWP[37], SCAN, DOUBLE SCAN[32], CIRCLES[33], etc. along with range-free localization method can be applied in pre-localization step such that all sensors have an estimate of their Initial Estimative Region (IERs). Even a less energy-intensive pre-localization process can be used where the mobile anchor is located at the center of square-like monitoring region and broadcasts beacons with different levels of transmission powers. So, in this case a circle, annular regions or overlap of an annular region with a square indicate IERs of sensors. Figure 2 illustrates an instance of pre-localization process by a mobile anchor equipped with a transceiver which has five different transmission levels. Since the mobile anchor consumes much more energy in movement process than beacon broadcasting, ignoring mobile anchor energy consumption in pre-localization step is rational.

3.2 The investigated problem
As the mobile anchor moves along the proposed path, it broadcasts beacons with fixed transmission power at all of the Predetermined beacon points including its current coordinate, $(x_b, y_b)$. This information is detected by the covered sensor nodes and is used to update their location information via construction of a new estimative region. Therefore, the receiver sensor confines its possible location to the area of intersection of the communication region of the mobile anchor at $(x_b, y_b)$ and its current estimative region. This process is repeated at the sensor nodes for each received beacon packet. Sensor nodes localize themselves based on beacons that they receive from the mobile anchor and their connectivity information. They impose geometrical constraints on their location estimations in order to reduce their location uncertainty. If $IER_i$ shows the initial estimative region of the $ith$ sensor, $S_i$, and $CR(b_j,r)$ in Equation (1) shows the communication region of the mobile anchor at the $jth$ selected beacon point, then $NER_i$ in Equation (2) is the new estimative region of the covered sensor, $S_i$. So, the area size of $NER_i$ shows the location uncertainty of sensor $S_i$ after receiving.
Finding the optimal trajectory of mobile anchor for sensor beacon from mobile anchor at $b_j$.

$$CR(b_j, r) = \left\{x, y \mid (x - x_{b_j})^2 + (y - y_{b_j})^2 \leq r^2\right\}$$

(1)

$$NER_i = IER_i \cap CR(b_j, r)$$

(2)

If the area size of the new estimative region $NER_i$ is smaller than that of the initial estimative region $IER_i$, then the location uncertainty of sensor $S_i$ is improved and the received beacon is considered to be an effective one. Otherwise, it is an ineffective beacon which only wastes the sensors’ energy. However, this ineffective beacon for sensor $S_i$ may act as an effective beacon for one or more other sensors. Figure 3 illustrates an instance of the construction of $NER_i$ when sensor $S_i$ receives an effective beacon in Figure 3(a) and an ineffective beacon in Figure 3(b). It may be concluded that if location uncertainty improvement of sensor $S$ indicates its localization success, then the number of received effective beacons and their origin’s distance from the sensor $S$ are critical decision making factors in mobile anchor path planning for range-free localization.

It is not pragmatic to consider that the position of deployed sensors don’t change and only one time mobile anchor-assisted localization is sufficient to provide sensed data with valid location information. The position of the sensors, may randomly change due to unintentional environmental factors such as animals, wind, etc. especially in outdoor environments. Moreover, the mobile anchor gradually reduces the location uncertainty of sensors each time it transmits an effective beacon. Hence, mobile anchor lifetime considered as a critical factor in mobile anchor path determination.

Consequently, the main challenge addressed in this study for the optimal path planning is to obtain maximum localization success while considering an energy constraint for the mobile anchor at the presence of sensors with known location uncertainty.

3.3 The proposed path planning approach

Finding the optimal trajectory of mobile anchor for sensor network localization is a very challenging problem. However, path planning for this particular application is done based on two criteria: (a) to offer full network coverage and (b) to determine good quality beacon points. However, the first criterion becomes less rigid due to energy constrained mobile anchor in the case being considered here. Instead, this paper aims at ensuring localization of the maximum number of possible sensors under priority-based optimization policy.

Definition of the second criterion is more challenging. Localization type, range-free or range-based, is a determinative factor of good quality beacon point definition. In this paper, where range-free localization is utilized, we consider a beacon point to be of good quality if it can at least improve the uncertainty of one sensor location. Otherwise, it will be considered to be an incompetent beacon point.

The novel mobile anchor path planning mechanism presented in this section is called Optimal Priority Based Trajectory planning with Energy Constraint (OPTEC) where the Mixed Integer Linear Programming (MILP) optimization approach is used to select optimal beacon points based on their appointed priorities in the context of sensors with known estimative regions.

It also considers the predetermined energy budget of the mobile anchor as a constraint in the optimal path determination problem. The key motivation for imposing this constraint is to extend the mobile anchor’s lifetime which is essential for localization of static sensors in outdoor environments with a high likelihood of dynamic behavior.

If three beacon points form an equilateral triangle, the location uncertainty of the covered sensor by these three beacons is the minimum. The mobile anchor path which is built based on this concept is called LMAT [34]. Therefore, the OPTEC path planning algorithm constructs the optimal path through the LMAT candidate beacon points shown in Figure 4.

The proposed path planning mechanism consists of two phases as described below:

Phase 1: Priority Definition

$$LIU_{S_i}$$

$$NER_i$$

$$CR(b_k, r)$$

Fig. 5. Priority definition for two different beacon locations due to sensor $S_i$: (a) beacon reception from $b_1$ is more probable than receiving a beacon from $b_2$. (b) beacon broadcasting at $b_2$ will contribute more in $LIU_{S_i}.$
Before running the \textit{OPTEC} algorithm, an offline priority calculation phase is necessary. In this phase, a priority table that includes the priorities of all candidate beacon points which are stored at the mobile anchor is constructed. In this case, a vital research issue is how to prioritize candidate beacon points in order to construct a befitting trajectory which can maximize the number of the sensors covered and the amount of location uncertainty improvement of the covered sensors.

Let $LUI_{S_i}(b_j)$ indicates the amount of location uncertainty improvement of $S_i$ covered by mobile anchor at beacon point $b_j$ defined in Equation (3) as follows:

$$LUI_{S_i}(b_j) = \text{Area}(IER_i) - \text{Area}(NER_i)$$  \hspace{1cm} (3)

If sensor $S_i$ is located in common region of $IER_i$ and $CR(b_j, r)$, it will experience location uncertainty improvement of $LUI_{S_i}(b_j)$ after receiving the beacon broadcasted at $b_j$. Let $\text{Area}(IER_i)$ be the area of Initial Estimative Region for $i$th sensor. Let $\text{Area}(NER_i)$ be the area of region from $IER_i$ that only if $i$th sensor located in that region will receive beacon broadcast at $b_j$. Then, $\text{prob}_{S_i}(b_j)$ in Equation (4) denotes the probability of sensor $S_i$ coverage by mobile anchor when it broadcasts beacon at $b_j$.

$$\text{prob}_{S_i}(b_j) = \frac{\text{Area}(NER_i)}{\text{Area}(IER_i)}$$  \hspace{1cm} (4)

This Equation is explained more by an example in Figure 5. In this figure solid elliptical regions indicate $IER_i$ which are the same size and shape for both Figures 5(a) and (b). Dashed circles are Communication Regions($CR$) of mobile anchor at beacon points $b_1$ and $b_2$ with equal radius. Red-colored regions show $NER_i$ whose $i$th sensor receives a beacon from beacon location, $b_1$ or $b_2$. This figure shows if $i$th sensor is being located in red-colored region ($NER_i$), it will receive the broadcasted beacon otherwise it won’t be covered by this beacon point. Due to bigger $NER_i$ in figure 5(a) than figure 5(b), it is more probable that $i$th sensor receive broadcast beacon form $b_1$ than $b_2$.

The probability of sensor coverage by mobile anchor at a given beacon point and the resultant location uncertainty improvement value define the priority of the candidate beacon point $b_j$ due to covered sensor $S_i$ in Equation (5).

$$P_{S_i|b_j} = \text{prob}_{S_i}(b_j) \times LUI_{S_i}(b_j)$$  \hspace{1cm} (5)

Finally, the priority of the candidate beacon point due to all its covered sensors is indicated by $P_{b_j}$ in Equation (6). Where $N$ indicates total number of sensors deployed in the monitoring environment.

$$P_{b_j} = \sum_{i=1}^{N} P_{S_i|b_j}$$  \hspace{1cm} (6)

The definition of priority of each beacon point in this work is affected by two criteria. (i) the number of the sensors that it covers and (ii) the resultant priority value due to their coverage by the beacon point. As Equation (5) shows, the probability of sensor coverage and its improvement on location uncertainty, which can potentially be in conflict with each other, are determinant in the value of the second criterion. Figure 5 shows examples of priority calculation at two different beacon locations resulting in different contribution for location uncertainty improvement. As shown in this Figure, the $IER_i$ whose boundary is marked by a solid elliptical region is of receiving the beacon broadcasted at $b_1$ is more than $b_2$, at the cost of smaller location uncertainty improvement.

Phase 2: Optimization Problem Formulation

In this step we give a more detailed description of decision variable, cost function and constraints to formulate the optimization problem as a Mixed Integer Linear Program(MILP). The \textit{OPTEC} problem is modeled as a fully-connected graph $G(v, \xi)$, where LMAT-based candidate beacon points $b_k$ are its vertices and $\xi$ represents direct paths between candidate beacon points. The maximum value for location uncertainty improvement of the sensors is achieved if the mobile anchor travels over all the LMAT-based candidate beacon points in the monitoring region. However, only a limited number of LMAT-based candidate beacon points are selected to construct the mobile anchor path due to the energy constraint. This section aims at the determination of optimal beacon points and building the shortest mobile anchor trajectory. Assuming $\xi_{ij}$ is the edge between the vertices $b_i$ and $b_j$ and $R_{b_i}$ and $R_{b_j}$ indicate the calculated priorities of $b_i$ and $b_j$, respectively, and then $TLUI_{WSN}$ is defined as the total localization uncertainty improvement achieved, where $M$ is the total number of candidate beacon points is shown in (7).

$$TLUI_{WSN} = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} ((P_{b_i} + P_{b_j}) \cdot \xi_{ij})$$  \hspace{1cm} (7)

The mobile anchor path optimization problem aims at maximizing $TLUI_{WSN}$ is formulated in Equation (8):

$$\text{Max } TLUI_{WSN}$$  \hspace{1cm} (8)

Subject to:

$$\sum_{i=1}^{M-1} \xi_{ij} \leq 1 \hspace{1cm} \forall \ j = 2, ..., M$$

$$\sum_{i=2}^{M} \xi_{ij} \leq 1 \hspace{1cm} \forall \ i = 1, 2, ..., N$$

$$\sum_{i=1}^{M} \xi_{ij} \leq \sum_{k=j+1}^{M} \xi_{jk} \hspace{1cm} \forall \ j = 2, 3, ..., M - 1$$
\[
\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \left( (x_i - x_j)^2 + (y_i - y_j)^2 \times E_d + E_{BR} \right) \times \xi_{ij} \leq E_{MA}
\]

where \(E_d\) is the mobile anchor energy consumption value to move one meter and \(E_{BR}\) is the transmission energy that the mobile anchor consumes to broadcast a beacon packet, \(E_{MA}\) is the energy budget of the mobile anchor for localization task, and \((x_k, y_k)\) is the coordinate of the \(k\)th candidate point. The first and the second constraints insure that the mobile anchor broadcasts a maximum of one beacon at each candidate point. The third constraint guarantees the connectivity of the subgraph which shows the optimal resultant path. The last one indicates that the total energy consumption of the mobile anchor to traverse the path and broadcast beacons should be equal to or smaller than its energy budget \((E_{MA})\).

In this paper MILP is applied to achieve the best outcome called \(OPTEC\) trajectory since the optimization problem that is described in Equation (8) includes a linear cost function, TLU_{WSN}, subject to linear equality and linear inequality constraints.

MILP is a non-convex optimization problem that can be solved by a heuristic, systematic or potentially exhaustive search. An integer linear program is a linear program that is further constrained by the integer outputs. The solution begins by solving an initial relaxed (non integer) problem using linear programming solutions. This paper uses the Branch and Bound method [42] to choose non-integer solutions of the relaxed problem. Then creates two new sub problems in which the value of that variable is more tightly constrained. These sub problems are solved and the process is repeated until a solution that satisfies all of the integer constraints is found. Finally, \(D = \{\xi_{ij} | \xi_{ij} = 1\}\), is set of edges that construct \(OPTEC\) trajectory, where the decision variable \(\xi_{ij} = 1\), indicates that the edge between \(b_i\) and \(b_j\) is part of the \(OPTEC\) subgraph and \(\xi_{ij} = 0\) otherwise. \(B = \{b_i, b_j | \xi_{ij} = 1\}\) shows the optimal selected beacon points. The optimal trajectory is then determined offline and stored at the mobile anchor beforehand.

4. Performance evaluation

This section compares the performance results of the proposed Optimal Priority based Trajectory with Energy Constraint (OPTEC) with four different existing trajectory planning approaches, namely SCAN [32], Z-curve [30], LMAT [34] and RW [37] algorithms under different evaluation metrics using Matlab.

In this section, the evaluation metrics are defined first, and then the simulation setup and the employed wireless channel and the related parameters will be described. Moreover, the simulation results are generated based on the defined evaluation metrics and they are discussed in detail.

4.1 Evaluation metrics

4.1.1 Localization coverage ratio

In this paper localization coverage implies the percentage of deployed sensors that have experienced location uncertainty improvement in the mobile anchor-assisted localization task. In other words, it shows the percentage of deployed sensors that receive at least one effective beacon while the mobile anchor moves along the trajectory.

4.1.2 Localization success

The resultant improvement on location uncertainty of sensors is the strongest metric for evaluation of mobile anchor trajectory planning in a range-free localization method. This paper defines the localization success for each sensor as the ratio of its location uncertainty improvement as shown in Equation (9)

\[
LS_i = \left(1 - \frac{LU_i}{Area(IR_i)}\right) \times 100
\]

where the location uncertainty of a sensor \(S_i\); denoted as \(LU_i\) shows the area size of its resultant estimative region \((RER_i)\) after the localization task. In the best case, \(RER_i\) is a point implying that \(LU_i\) is zero. In the worst case, sensor \(S_i\) does not receive any effective beacon, and the area size of its estimative region does not change.

Therefore, localization success of WSN is measured by averaging the localization success of the sensors as shown in Equation (10), where \(N\) indicates the number of deployed sensors.

\[
LS_{WSN} = \frac{\sum_{i=1}^{N} LS_i}{N}
\]

4.1.3 Mean location uncertainty

In range-free localization, the smaller area size of \(RER_i\) indicates that the mobile anchor broadcasts beacon at more effective beacon points. So, it is more fair to interpret \(LU_i\) as the localization error of \(S_i\). In this case, localization error is measured regardless of the method used for position estimation. \(LU_{WSN}\) in Equation (11) is used in this study as an impartial metric to evaluate static trajectories. \(N\) shows the number of deployed sensors.

\[
LU_{WSN} = \frac{\sum_{i=1}^{N} LU_i}{N}
\]

4.1.4 Mean Location Error

In many existing applications of WSNs, the estimative region of the sensors obtained by coarse-grained node localization is useless and the exact position of sensors is required. In these cases, the estimative region of the sensors should be converted to a location coordinate estimation. In this work, the center of gravity of the sensors estimative region is interpreted as their location estimate.

In such applications, location error is an essential metric to evaluate the accuracy of the mobile anchor trajectory. In this
paper, the location error of sensor $S_i$ is defined in the form of Root Mean Square Error (RMSE) as follows:

$$L_{ei} = \sqrt{ (x_{ei} - x_{Ri})^2 + (y_{ei} - y_{Ri})^2 }$$

(12)

Where $(x_{ei}, y_{ei})$ and $(x_{Ri}, y_{Ri})$ show the estimated location coordinates and the real location coordinates of $S_i$, respectively. Therefore, the mean location error of all deployed sensors in Equation (13) is representative of the accuracy metric in the applications that need the exact location of the sensors.

$$L_{ewsn} = \frac{\sum_{i=1}^{N} L_{ei}}{N}$$

(13)

### 4.2 Simulation setup

In the scenario being investigated here, stationary sensor nodes with given IERs, are deployed randomly and uniformly over a two-dimensional square monitoring environment area of 100m×100m. Furthermore, an energy-constrained mobile anchor is moving around and does the range-free localization task.

#### Wireless Link Budget Analysis

In order to have reliable evaluations, a detailed site survey and wireless link design are important. In this section, we consider channel model, modulation, and the desired BER in order to obtain the relation between the communication radius and the transmission power. Some of the major factors that determine the communication radius are transmission power, antenna gains, propagation path loss and the minimum sensitivity of the receiver. So the following equation shows the elements required to calculate the communication radius.

$$P_{RX} = P_{TX} + G_T + G_R - P_L$$

(14)

where $P_{RX}$ shows the receiver power (which is substituted by receiver sensitivity to calculate the communication radius), $P_{TX}$ is the transmitter power, $G_T$ and $G_R$ are the transmitter antenna gain and the receiver antenna gain, respectively and $P_L$ shows the path loss.

Receiver sensitivity is important. That is because it guarantees the required signal strength at the receiver input. It depends on two main factors: The signal-to-noise (SNR) ratio at the receiver output and the receiver noise floor as defined in the following equation:

$$\text{sensitivity} = SNR_{out} + \text{Receiver Noise Floor}$$

(15)

$SNR_{out}$ is the minimum $SNR$ that guarantees the desired bit error rate (BER). At the output of the receiver, the $SNR$ is calculated by (16).

$$SNR_{out} = \frac{E_b}{N_0} \times \frac{R}{B}$$

(16)

Where $\frac{E_b}{N_0}$ is a measure of the required energy per bit relative to the noise power, which is extracted from the graph of $\frac{E_b}{N_0}$ vs BER for a given modulation technique. $N_0$ is the thermal noise in 1Hz of bandwidth. $R$ is the data rate, and $B$ shows the receiver bandwidth. Real receiver noise floor shown in (17) will always be higher than thermal noise($KTB$), due to noise and losses in the receiver itself. Noise Figure (NF) is a measure of the amount of noise added by the receiver itself.

$$\text{Receiver Noise Floor} = 10 \log KTB + NF$$

(17)
where $K$ is the Boltzmann’s Constant, $T$ is the absolute temperature and $B$ shows the bandwidth.

The generic path loss model in Equation (18) describes the signal attenuation between a transmitter and a receiver antenna as a function of the distance and other parameters.

\[
P_L = 10 \log \left( \frac{A d f}{c} \right)^\gamma = 10 \gamma \log d + 10 \gamma \log f + 10 \gamma \log \left( \frac{A}{c} \right)
\]

where $d$ is the distance in meters, $C$ is the speed of light, $\gamma$ is the pass loss exponent and $f$ is the frequency in Hz.

Here we consider TElosB mote which is equipped with an IEEE 802.15.4 compliant chipcon CC2420 radio [43]. Thus, we use Equation (19) for $P_L$ as suggested in the of annex E of [44], in order to calculate path loss for 802.15.4-compliant systems.

\[
P_L = 33 \log \frac{d}{\delta} + 58.5
\]

Simulation parameters used in this paper are listed in Table 1.

4.3 Simulation results

Extensive simulations were carried out using Matlab. Performance comparison of four existing mobile anchor path planning mechanisms, i.e. SCAN [32], LMAT [34], RW [37], Zcurve [30], and the proposed method (OPTEC), is done using the above-mentioned evaluation metrics. In order to ensure a reliable comparison for each experiment, monte Carlo simulations were run for 100 times and in each run, a new random uniform deployment of sensors was used.

The evaluation results are illustrated and explained in the following subsections. In this set of simulations, we assume that the total required energy for the mobile anchor to traverse the entire network is divided into 100 energy units.

4.3.1 Mobile anchor trajectory efficiency

We consider both the localization coverage and the localization success as the critical metrics to analyze the efficiency of the path planning mechanisms.

**Localization Coverage:** The localization coverage is shown in Figure 6 under different values of available energy for the mobile anchor. The performance of investigated trajectories in terms of localization coverage is independent of the number of beacons received by each sensor. It is only defined based on the fact that localizable covered sensors receive at least one beacon which leads to their location uncertainty improvement. Regardless of the available energy of the mobile anchor, OPTEC outperforms the other trajectories. The values of localization coverage trajectory improvements due to the OPTEC in comparison with the best existing trajectory are also shown in Figure 6. It is observed that the localization coverage improvement decreases as the available energy of the mobile anchor increases. Hence, the advantage of the OPTEC in the localization coverage is more impressive for lower values of available energy.

**Localization Success:** Localization success of the trajectories are studied under the constraint of energy consumption for the mobile anchor movement as depicted in Figure 7. According to the definition mentioned in Equations 9 and 10, localization coverage, the number of localizable sensors experience location uncertainty improvement and their location uncertainty improvement values determine localization success. Figure 7 shows OPTEC performs better than the other existing static trajectories. As depicted in this Figure, it can significantly improves the localization success up to 31%,16% and 6% at 30%,60% and 90% of the mobile anchor available energy, respectively. This behavior is interpreted by the fact that the OPTEC trajectory is constructed among the optimal beacon points considering the mobile anchor energy constraint.

4.3.2 Localization accuracy:

As a conclusion for the accuracy achieved by the various trajectories, we analyzed both the mean location uncertainty and the mean location error of the sensors under different energy values for the mobile anchor. Moreover, we have performed this set of simulations under different network densities to study its effect on localization accuracy improvement value due to the OPTEC trajectory.

**Mean Location Uncertainty:** The location uncertainty is divided by the mobile anchor communication circle area to define the location uncertainty ratio. Figure 8(a) measures the impact of the consumed energy of the mobile anchor on the location uncertainty ratio of the deployed sensors. The location uncertainty reduction with the increase in energy consumption is a common feature of all trajectories. However, the proposed trajectory OPTEC outperforms the other mechanisms in terms of the location uncertainty ratio in all cases of energy consumption. The major reason is that the proposed approach guides the mobile anchor along the other mechanisms in terms of the location uncertainty ratio in all cases of energy consumption. The major reason is that the proposed approach guides the mobile anchor along the most promising LMAT-based beacon points which are selected based on their assigned priority value. This comparison is done regardless of the localization method used in order to determine the coordinate of the sensors. Therefore, it is a fair metric to examine the performance of the trajectories in a range-free localization process. Due to the impartiality of the static trajectories in localizing sensors, we consider standard deviation of the location uncertainty. A smaller value of the standard deviation
Table 2 Mobile anchor energy and localization time needed in path planning mechanisms for range-free localization approach with different values of ALU ratio (The velocity of MA is set to $v = 2 \text{ m/sec}$)

<table>
<thead>
<tr>
<th>Mean Location Uncertainty Ratio ≤ ALU</th>
<th>Trajectory</th>
<th>Energy Consumption(J)</th>
<th>Localization Time(sec)</th>
<th>Average Localization Time per localized sensor(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>OPTEC</td>
<td>19.11</td>
<td>93.2</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>LMAT</td>
<td>22.3</td>
<td>118.68</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>SCAN</td>
<td>23.9</td>
<td>119.1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Zcurve</td>
<td>25.48</td>
<td>127.17</td>
<td>4.24</td>
</tr>
<tr>
<td></td>
<td>RW</td>
<td>41.4</td>
<td>221.3</td>
<td>6.62</td>
</tr>
<tr>
<td>0.5</td>
<td>OPTEC</td>
<td>28.67</td>
<td>146.6</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>LMAT</td>
<td>35.04</td>
<td>178.22</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td>SCAN</td>
<td>38.22</td>
<td>184.03</td>
<td>3.06</td>
</tr>
<tr>
<td></td>
<td>Zcurve</td>
<td>47.7</td>
<td>237.2</td>
<td>4.90</td>
</tr>
<tr>
<td></td>
<td>RW</td>
<td>95.57</td>
<td>493.37</td>
<td>8.92</td>
</tr>
<tr>
<td>0.4</td>
<td>OPTEC</td>
<td>44.6</td>
<td>224.8</td>
<td>2.74</td>
</tr>
<tr>
<td></td>
<td>LMAT</td>
<td>52.56</td>
<td>265.89</td>
<td>3.39</td>
</tr>
<tr>
<td></td>
<td>SCAN</td>
<td>54.16</td>
<td>259.8</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Zcurve</td>
<td>71.68</td>
<td>355.92</td>
<td>5.36</td>
</tr>
<tr>
<td></td>
<td>RW</td>
<td>159.28</td>
<td>811.06</td>
<td>11.36</td>
</tr>
</tbody>
</table>

indicates that the sizes of the resultant estimative regions of all the sensors are likely to equal. It is defined as follows:

$$\text{Std}d_{LU} = \sqrt{\frac{\sum_{i=1}^{N}(LU_{Si} - LU_{WSN})^2}{N}}$$ \quad (20)

As it is expected, it is obvious from Figure 8(b) that as the mobile anchor energy is increased, the standard deviation of location uncertainty is decreased. It is concluded from Figure 8(a) and 8(b) that regardless of the mobile anchor energy consumption, any of the predetermined static trajectories perform better than RW in terms of location uncertainty and the OPTEC has the lowest standard deviation of location uncertainty compared with other static path planning mechanisms.

To evaluate the accuracy achieved by the investigated trajectories when employing range free localization, we defined a new parameter called Acceptable Location Uncertainty ratio (ALU ratio). The ALU ratio is a user predefined value. The smaller value of ALU ratio indicates that the estimated region of the deployed sensors should be reduced more after the localization task. We summarized the consumed energy of the mobile anchor and the localization time spent to meet this criterion in Table 2. As it is observed, the OPTEC trajectory consumes less energy than the other trajectories to meet the requirement of a given expected location uncertainty ratio. The proposed trajectory also has the lowest value of total localization time and average localization time per localized sensor. When the ALU ratio equals to 0.6, the OPTEC saves energy and localization time by 14% and 21%, respectively in comparison with the best existing trajectory.

**Mean Location Error:** In location dependent applications where the exact coordinates of the sensors are needed, mean location error can be used to judge the performance of the mobile anchor based localization processes. Let the mean location error be defined by the average square of the distance from the center of gravity of all the sensors’ estimated regions to their real location. In this case, the mean location error ratio is calculated when the mean location error is divided by the mobile anchor communication radius. Figure 8(c) compares the mean location error ratio of five path planning schemes in each of the mobile anchor energy constrained cases. Increasing the mobile anchor available energy improves the localization accuracy for all of the investigated schemes. It can be seen that the mean resultant location error ratio under the OPTEC is lower than that achieved by any of the other trajectories. By contrast, the RW results in high error value in comparison with the other schemes. For instance, when the energy of the mobile anchor is equal to 70%, the mean location error ratio of OPTEC is only 38.36%, whereas those of the LMAT, SCAN, Zcurve and the RW schemes are 40.45%, 42.11%, 46.36%, and 49.81%, respectively.

**The Impact of Sensor Nodes Density on Location Error:** Figure 8(d) measures the impact of sensor node density and the energy spent for the localization on the mean location error ratio. When the network density is fixed, the mean
According to the application of WSNs in hazardous and inaccessible environments, WSN lifetime which has a strong dependence on the energy consumption of sensor nodes is the main challenge. Hence most of the existing research works have focused on energy conservation techniques. In this section, we evaluate trajectories with regards to the energy consumed by the sensor nodes in localization process and ineffective beacons have been broadcasted by the mobile anchor.

**Energy Consumption of Sensors:** Unknown sensors consume energy whenever they receive a beacon packet from the mobile anchor. Therefore, their energy consumption depends on the number of received beacon packets and their size in bits. Furthermore, the power consumption of RF transceiver in the receive mode and its effective data rate are significant factors in determining the energy consumption value of the sensors. Thus, the total energy expended per localized sensor can be calculated by:

$$E_{sensor} = N_R \cdot E_{rx}$$

$$E_{rx} = P_{rx} \cdot \frac{\text{beacon packet size}}{\text{data rate}}$$

Where $N_R$ is the average number of received beacons by each localized sensor and $E_{rx}$ denotes energy consumption per beacon packet. Based on TElosB mote which is equipped with an IEEE 802.15.4 compliant chipcon CC2420 radio [39] current consumption for receive mode is 18.8 mA.

To provide a more reliable evaluation of performance analysis, we examined the energy spent by each localized sensor at the expense of localization success which is plotted in Figures 9(a). As this Figure shows, SCAN outperforms the others in terms of energy consumption per localized sensors. However Zcurve consumes more energy than the other static trajectories. This figure shows the superiority of the OPTEC scheme while it provides 9.5% improvement in localization success. However, the energy efficiency of SCAN is best if we take the energy consumption of the sensor nodes into account. So, a trade-off is observed between sensor nodes energy consumption with location uncertainty and also localization success. Although sensor nodes have the ability to trade energy for location uncertainty improvement, it is quite difficult to
improve both energy efficiency and accuracy simultaneously. Therefore, we define the Energy-Location uncertainty product (ELuP) as a figure of merit correlated with energy efficiency and location accuracy of sensor nodes. It is the product of energy consumption of localized sensors (averaged over a localization process) and the mean location uncertainty (defined location accuracy) of sensor nodes.

As Figure 9(b) shows, SCAN outperforms the other trajectories for \( E=80\% \) and \( E=90\% \). However, most of the time OPTEC has better performance than the others. In OPTEC, the main concern is to select optimal beacon points. This optimal selection results in a reduction of ELuP.

**Ineffective Beacon Points Rate:** In spite of the fact that more received beacons result in higher percentage of localization success, we introduce a new related metric that analyzes the inefficiency of beacon points in the investigated trajectories. Indeed ineffective beacon points explain the number of beacon points whose communication circle area is larger than the current estimative region of the receiver sensor or the number of beacon points whose broadcasted beacons are not received by any sensors. So, they are helpless in location uncertainty improvement of sensors. Ineffective beacon points rate (IBR) is given by Equation (22).

\[
IBR = 1 - \frac{\text{# of ineffective beacon points}}{\text{total number of beacon points}}
\]

(22)

Useless beacon points increase the energy consumption of sensor nodes while they do not have any contribution to the progress of the localization task. Our concern in OPTEC is the reduction of ineffective beacon points rate by assigning priority to candidate beacons based on their total contribution in uncertainty improvement of all sensors. Therefore as it is observed from Figure 9(c), the proposed trajectory surpasses the other static trajectories for all values of the mobile anchor energy.

4.3.4 Communication range to resolution distance ratio:
This subsection investigates the performance of the OPTEC trajectory in terms of the impact of communication range to resolution distance ratio where length of equilateral triangle in LMAT-based trajectory defines the resolution distance, \( d \). Covered sensors, localization coverage and mean location uncertainty of sensors are evaluation metrics for OPTEC with different values of communication radius to resolution distance ratio. Covered sensors indicates the percentage of sensors that have received at least one beacon. It can be deduced from Figure 10(a) that increment of communication radius to resolution distance ratio produces a significant growth in the number of covered sensors in all energy cases. However, it is not true for localization coverage shown in Figure 10(b). We observe that the highest level of localization coverage provided by OPTEC at different values of mobile anchor energy occurs for different values of \( R/d \). For instance, it reaches its highest localization coverage in \( \frac{R}{d} = \frac{1}{2} \) for an initial energy of 58J. While the highest localization coverage is achieved for \( E=32J \) at \( \frac{R}{d} = 1 \).

Figure 10(c) plots the mean location uncertainty in terms of mobile anchor energy under various values of \( \frac{R}{d} \). Approximately, the proposed trajectory provides the worst localization accuracy under \( \frac{R}{d} = 2 \) for all values of the mobile anchor energy. This poor performance is due to the lower percentage of localization coverage achieved under this communication radius to resolution distance ratio. Furthermore, lower value of location uncertainty improvement due to larger communication area of the mobile anchor is another reason for poor performance under \( \frac{R}{d} = 2 \). It is observed from Figure 10(c) that the mobile anchor provides the lowest location uncertainty under different \( \frac{R}{d} \) values for various initial energies. OPTEC under \( \frac{R}{d} = 1 \) outperforms the other cases for energy values around and lower than 50%. However, the higher precision is achieved under \( \frac{R}{d} = \frac{1}{2} \) for most of the time.

1. Conclusion and future work

To the best of our knowledge, this is the first study that applies
Mixed Integer Linear Programming (MILP) optimization approach for optimal route planning of the mobile anchor in the presence of location uncertainty for deployed sensors. Beacon points and mobile anchor route are design variables. The objective function is a linear function of total sensors location uncertainty improvement and mobile anchor energy consumption is the main constraint. Firstly, candidate beacon points are determined based on LMAT trajectory and prioritized based on their contribution in the total location uncertainty improvement of the sensors. Then, the MILP optimization approach is utilized in order to find the best beacon points while considering a given energy budget for the mobile anchor. Through simulations, we showed that the proposed scheme outperforms well known mobile anchor trajectories such as LMAT, Zcurve, SCAN and RW in terms of trajectory efficiency, localization accuracy and sensor nodes lifetime. Based on the simulation results shown in section IV, communication radius adjustment and considering it as another design variable in static path planning optimization problem is planned for future work. Furthermore, the future work will investigate heuristics approaches such as Tabu search heuristic in [45] to solve the mobile anchor path planning optimization problem.

References


